Chapter Four. SYMBOLIC REPRESENTATION OF WATER SURFACE HEIGHT AND WATER DEPTH

In the previous two chapters I have demonstrated that water surface height, a time- and space-dependent variable in all water bodies, shows particularly complex spatial-temporal variation in estuaries. I have also noted that the difference between water surface height and ground surface (or "topographic") height defines the variable "water depth", and shown that variations in estuarine stage and depth profoundly influence many physical, chemical, and biotic processes in estuaries and their fringing tidal marshes. The significance of stage and depth has been especially noted along shorelines and over marsh plains, where the depth periodically drops to zero (or to less than zero if the depth of the water table below the marsh surface is considered). In particular, I have demonstrated that the structure and functioning of estuarine tidal marshes is largely controlled by the patterns of overbank flooding and drying and the associated patterns of subsurface saturation and drainage. Finally, I have shown that the literature on these phenomena is extraordinarily diverse, containing contributions from many disciplines and intellectual traditions.

Therefore, I conclude that characterizing, understanding, and predicting estuarine and tide marsh structure and function will require a concise, unambiguous, and comprehensive system of definitions and symbolic representation for height, depth, and other associated variables. In this chapter I will extend the discussion in the preceding chapters by illustrating some gaps or areas of conflict in current symbol and word usage, in part by presenting a detailed concordance of the notation used in standard works on tidal phenomena (Schureman 1958; Pugh 1987; Parker 1991a; Parker et al. 1999; Hicks 1989 in NOS 1999). In this concordance and following it, I also present an extensive series of suggestions for bridging these gaps and reconciling these conflicts. This is followed by a detailed discussion of my suggestions and their justification and application.

I. Rationale

The significance of rigorous terminology and notation for rigorous analysis of stage is clear from the discussion of the literature in Chapter Two, and from the misunderstandings and inconsistent conclusions that were shown to have resulted from casual use of terms.

As demonstrated in Chapters Two and Three and in Table 17, there is a wide variety of available approaches to characterizing water surface elevation. In particular, I noted in Chapter Two that water surface height or elevation is also known as “stage” in fluvial studies and as "water level” in most other disciplines, but that “water level” is not a precise phrase because the upper surface of natural waters is rarely level except in limited areas for limited times. I also noted the importance of recognizing and preserving conventional usage where appropriate. Therefore, “water level” is used at times in this thesis, but only when refering to a local and instantaneous variable.

In one recent proposal to standardize usage, Mitchum (1997) proposed a distinction between measurements collected by tide gauges [“along the boundary where the ocean meets the land”], which he refers to as “sea level”, and those measurements made by satellite altimeters, which he refers to as “sea surface height”. While I agree that it is prudent to distinguish measurements collected through different means, his suggestion poses several difficulties: it is not consistent with common usage; it is ambiguous regarding measurements collected by sea-floor pressure gauges or GPS-mounted buoys, etc.; and, through emphasis, it incorrectly implies that measurements collected along the shore line are in fact level. Therefore, I do not adopt this convention.

In another example, I noted in Chapter Two that most estuarine and marsh researchers implicitly conflate “tide” and “water surface height”, while most physical oceanographers recognize tides only as “the periodic rise and fall of the water resulting from gravitational interactions between Sun, Moon, and Earth” (Hicks 1989). To most oceanographers, therefore, differences between observed water levels and harmonic tidal predictions are not errors, but merely “residuals”, or consequences of phenomena outside their scope of study. To mariners and most estuarine and marsh scientists, in contrast, temporal variations in water level are significant regardless of the underlying causes, and therefore any differences between observations and tide table predictions are essentially “errors”. To this audience, at least, any improvements in water level or depth prognostication, such as effective incorporation of weather or stream flow forecasts, would be welcomed. Therefore, a simple semantic device for accommodating both groups is to consistently distinguish between “tidal prediction” and “water level forecasting”, since the precedent is well established for using “forecasting” for parameters that are not as reliably predictable as the astronomical motions that generate the tides.

In addition, in most estuaries, tides account for most of the temporal variation in stage at scales longer than a few seconds (i.e. those variations in stage not induced by wind-generated waves), yet water levels measured in an estuary not only differ from water levels predicted solely from harmonic tidal constants and correctors, but also from water levels measured in the estuary’s associated ocean[161]. While differences from the harmonic predictions are formally known as “residuals”, there is no equivalent accepted term for differences in stage between the ocean and sites within the estuary, so I will suggest “divergences” (see detailed discussion below). Residuals and divergences result partly from

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[161] As noted in Chapter One, the words “ocean” or “the sea” are used here to describe any water body large enough to generate significant tides.

site-specific dynamic responses to harmonic drivers (e.g. turbulence, standing waves, tidal bores, etc.), and partly from nonharmonic atmospheric, fluvial, and oceanic processes that interact with the moon- and sun-driven tides to determine estuarine water levels. Examples of non-tidal influences on water level that I have discussed include short-term local weather effects (storm surges, river discharge, barometric pressure, local wind, etc.); longer-term episodic oceanographic phenomena (ENSO = El Niño-Southern Oscillation, etc.); and influences that act on water level patterns over a very long time scale (changes in climate, sea level, basin configuration or bathymetry, etc.). Therefore, any acceptable system for characterization of spatio-temporal change in stage or depth must span multiple spatial and temporal scales and must clearly define frames of reference, even if no single reference plane is stable over all temporal and spatial scales of interest.

II. Concordance and Suggestions

Table 17 compares and contrasts the symbolic notation used in five standard works on tides, along with the conventions I use in this thesis. The table is divided thematically, starting with elevations and vertical datums and moving to related categories: general mathematical conventions, data collection and oceanographic parameters, time, astronomical constants and geometry, tidal harmonic constants, and currents and waves. Each section is alphabetical, with Greek letters following English. Lower case letter precede capitals, and letters with subscripts follow those without.

In the interests of clarity and consistency, I have followed conventional symbol usage to the extent possible, but inconsistencies between previous authors, overuse of some letters as symbols, ambiguous prior definitions, and my recognition that some important variables have no conventional notation have all led me to propose the system of notation and terminology that is described in the concordance and following it.

Table 17: List of Symbols and Concordance with Previous Authors

My suggested terms in Table 17 contain two significant departures from the current NOS “Tide and Current Glossary” (Hicks 1989 in NOS 1999): I retain the earlier use of Greek letters (e.g. Schureman), and, in particular, I propose that the capitalized Greek letter zeta (**Ζ**) be used to represent water surface elevation relative to fixed geodetic (orthometric/gravimetric) vertical datums. I make the general recommendation because there are simply not enough letters in English to represent all the variables commonly discussed in tidal hydraulics, as is clearly demonstrated by Table 17. Specifically, the letters “M, m” (moon, mean, mass, etc.); “L, l” (lunar, low, length, etc.); “S, s” (sun, spring, surge, etc.); “T, t” (time, tide, tropic, etc.); “H, h” (head, height, high, etc.); “D, d”(depth, declination, diurnal, derivative, etc.); “A, a” (amplitude, right ascension, diameter of the earth, etc.); and “P, p” (perigee, perihelion, pressure, etc.) are all overused.

The use of **Ζ** for water surface elevation relative to a fixed geodetic vertical datum is appropriate for a number of reasons. First, this is consistent with the common use of the English letter “Z, z” to represent vertical in three-dimensional Cartesian coordinate systems (Pugh 1989). Second, the lower case zeta (**ζ**) is widely used in hydrodynamic equations to represent water level relative to its “undisturbed” height (Pugh 1987; Parker 1999). Third, the concept of vertical zonation, which underlies much of the academic interest in tidal datums relative to coastal elevations, is summarized by the Greek word *Zōnē*, meaning zone or belt (OED 1991). Finally, neither **Ζ** nor the English **Z** (which it closely resembles in most fonts) have any other conventional usage in this discipline[162], minimizing any risk of ambiguity.

In addition to the use of Greek letters and Ζ in particular, my second major set of recommendations is summarized in Figure 55, which presents a suggested notation set for height (i.e. elevation) relationships between water levels, tidal predictions, tidal datums, tidal means, topographic/bathymetric surfaces, and geodetic datums (compare to Figures 6 and 7). Figure 55 and its components will be explained in the discussion following Table 17.

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[162] Other Greek letters that might be appropriate include Θ θ (theta, the first letter of *thalassa*, the sea), Υυ (upsilon, the first letter of *hudor* = hydro), or Ψ ψ (psi, which has symbolic associations with the trident of Poseidon) (Chris Simon, University of California; and Martha Malamud, SUNY Buffalo; pers. comm.). However, lower case theta and upsilon both have conflicting conventional usage, and none of these have the other positive attributes of Ζ.

Figure 55: Proposed Height (Elevation) Series at the Golden Gate

Finally, Figure 56 presents a third integrated set of recommendations – specifically, a suggested hierarchy of terms describing elements of the spatial variability of temporal patterns of water level (i.e. spatial coherence and variation in tidal regime). Other suggestions are incorporated in Table 17 and in the discussion that follows.

- **(Stage) Difference** = Absolute difference in stage between two points at time t
  - **Lag** = Mean time for HW to propagate from primary station (PS) to point XY
  - **Divergence** = Difference, corrected for HW Lag
    - **Mean Water Slope** = Increase in absolute elevation of mean wave stage
    - **Attenuation/Amplification** = Change in wave amplitude/range
    - **Distortion** = Changes in the harmonic wave form
      - **Steepening** = Progressive increase in ebb-flood asymmetry of stage vs time
      - **Deformation** = Other wave-form distortions
        - **Diking** = Elimination of tidal action by built structures
        - **Muting** = Reduction of tidal action, generally by built structures
        - **Thresholds** = Filtering by ground surface elevation
        - **Ponding** = Retention of water during low water
        - **Velocity Pulses** = Non-harmonic vertical or horizontal accelerations
        - **Stage-Dependent Flows** = Due to topography or structures
          - **Marsh Plain Discharge** = Over-bank influence on channel stage
          - **Pseudo-Tide** = Above tide gates
          - **Augmented Low Stage Flows** = Below tide gates
          - **Tidal Pumping** = Above culvert or other ebb-flow obstruction

Figure 56: Hierarchy of Terms Describing Spatial Variation in Tides

The terms in Figure 56 were carefully selected for clarity, for consistency with their usage in common English as well as in technical fields, and to avoid conflicts with previously defined terms. For example, the Oxford Universal Dictionary (1989) gives similar definitions for “distortion” and “deformation”, but “deformation” has a more strongly pejorative sense than “distortion” and is hence used for the more “monstrous” alterations in form. “Deviation”, which has a definition similar to that of “divergence”, is not included in Figure 56, and is reserved for differences in wave forms from harmonic ideals rather than from actual observations at another location. Finally, “difference” is a commonly used word, but “water level difference” or **$\Delta Z(t)$** should be unambiguous. Only “augmentation”, “deviation”, and “lag” occur both in this discussion and in the current NOS Glossary, and of these only “lag” could possibly lead to confusion. To avoid this, I will generally use **ΔHWI** for “lag” as I define it in Figure 56.

III. Discussion

Water, like any other liquid[163], fills the shape of its container up to a “free surface” at the water body’s upper boundary. In this section I discuss, justify, and apply a rigorous system for characterization of this free surface and, in particular, of the spatio-temporal variation of its elevation. I begin with general principles and then move to the specific case of estuarine stage and tides. In addition to a general system of notation to describe water surfaces, I discuss a new synthetic approach to indicating height relationships (Figure 55) and a new nested hierarchy of term to describe changes in tidal waves as they propagate through estuaries (Figure 56).

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[163] The special cases of ice or ice/liquid water mixtures are not discussed in this paper.

A. Representation of Water Surface Elevation

The upper boundary of any body of liquid water on the earth is a spatially-continuous, approximately-horizontal surface that can be described in functional notation by $\mathbf{Z(x,y,z_0,t)}$, where $\mathbf{Z}$ is water surface elevation, $\mathbf{x}$ and $\mathbf{y}$ are spatial coordinates in the horizontal plane (perpendicular to the acceleration of gravity[164]), $\mathbf{z_0}$ is a vertical datum (any horizontal plane), and $\mathbf{t}$ is time. The surface is spatially-continuous because of surface tension, which characterizes liquid water as a liquid, and is approximately horizontal because of the force of gravity and the great difference in density between liquid water and air. When high-frequency and generally moderate-amplitude wind waves are filtered out, deviations from horizontality in estuarine water surfaces, which form much of the subject matter of this thesis, are geometrically minor, rarely exceeding 1% slopes. These minor slopes, however, correspond to elevation differences of great functional significance[165].

At any $\mathbf{x,y}$ location in any water body, the elevation of the water surface relative to any vertical datum is a function of time, and may be represented by $\mathbf{Z_{XY}(z_0,t)}$ or simply $\mathbf{Z(z_0,t)}$ (= “stage relative to vertical datum $\mathbf{z_0}$ at time $\mathbf{t}$”). When the vertical datum is explicit and consistent (e.g. NGVD29; “local MLLW defined by the time period 1/1/1960-12/31/1978"; etc.), the shorthand $\mathbf{Z_{z0}(x,y,t)}$ or simply $\mathbf{Z(x,y,t)}$ can be used rigorously. If $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z_0}$ are all defined explicitly and consistently, $\mathbf{Z(t)}$ unambiguously represents the water level at a

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[164] For convenience, x and y are represented as mutually-perpendicular planar Cartesian coordinates, but the non-Euclidean nature of the Earth’s surface means that this approximation is not strictly accurate, especially for large water bodies. At the spatial scale discussed in this thesis, this approximation works well. At larger scales, spherical surfaces and coordinates are required, but all conclusions are valid. In addition, there is no need for x and y to be mutually perpendicular; indeed any non-colinear coordinate sets can define a location in the “horizontal” plane (or spherical surface) perpendicular to the acceleration of gravity.

[165] In general, the approximate horizontality of the surface results in a one-to-one correspondence between time and stage. The special case of breaking waves, in which the free surface can include 3 elevations for each x,y pair, is not discussed in this paper.

location through time. Where the vertical datum for an **x,y** location is the elevation of a stable and clearly defined surface at the bottom of the water column ($\mathbf{z_{B,XY}}$), the water depth $\mathbf{D_{XY}(t) = Z_{XY}(t) - z_{B,XY}}$ is a rigorous alternative, at least for the time period during which the substrate neither accretes nor erodes.

Because time is a continuous (non-discrete) variable, **Z(t)** is also a continuous variable. However, time-varying quantities can be efficiently and conveniently described by measuring them at consistent time increments, and $\mathbf{Z(t_n)}$ is a useful notation for a single measurement (the "nth") in the resulting time series. Water stage (or any other continuous function of time) can be described and represented to any arbitrary precision by sampling at high enough frequency (short enough intervals) relative to the changes in stage (or other variable). NOS typically records stage every 6 minute (= 0.1 hr), and each record consists of the arithmetic mean of 720 samples at 0.5 second increments, reflecting a judgement that tidal scale changes in stage can occur within six minutes, but not within half a second. In this protocol, $\mathbf{t_{n+1}} = \mathbf{t_n} + 0.1\text{hr}$. Alternatively, **Z(t)** can be represented by its tidal-scale maxima and minima ("high tides" = "high water" = "**HW**"; "low tides" = "low water" = "**LW**") by converting a continuous measurement of stage at a location into a record of the times and magnitudes of local maxima and minima (higher or lower, respectively, than any other measurements over a several hour period).

Any time series for any variable will demonstrate extreme values and mean values, and their interpretation will depend both on the data sampling interval and on an evaluation of which temporal variation is significant and which represents noise. In the case of tidally-influenced settings, the convention is to define HW and LW on the basis of the dominant tidal periods of one-half and one lunar days. Because the precise timing of tidal extremes at a site cannot be predicted a-priori (especially for a relatively new data collection site), measurements of **$Z(t)$** must be made continuously (as with a pen-trace) or at least at high frequency. This, in turn, requires data analysis techniques that filter out wakes, wind waves, and other noise. Because high frequency noise can generate strings of apparent maxima or minima in small time periods, NOS uses the averages of a series of measurements at 0.5 second intervals to smooth their data and determine HW and LW times and magnitudes (Jim Hubbard, NOS, pers. comm.). Other algorithms to reduce noise are also possible.

Because spatial location is also a continuous variable, **$Z_t(x,y)$** defines the instantaneous form of the water elevation as a continuous surface, with a one-to-one mapping from xy locations to stage. Therefore, if **L** is defined as the distance between two horizontal locations (XY and PS) in a water body[166], then **$(Z_{XY}(t) - Z_{PS}(t))/L$** = **$\Delta Z_{XY,PS}(t)/L$** represents instantaneous water surface slope, which indicates hydrostatic force and therefore potential lateral movement of water[167]. As with time, xy locations can be treated continuously or discretely. Discrete depictions of space can invoke either evenly spaced grids (typically used in finite-element hydrodynamic models) or particular locations of interest (typically used in tide tables). Spatial variability of **$Z(t)$** within an ocean, estuary, channel, or marsh can be either significant or insignificant for a given purpose. Where it is insignificant, notations such as **$Z_O(t)$, $Z_E(t)$, $Z_C(t)$**, and **$Z_M(t)$** (mean water levels at time **t** in an adjacent Ocean or

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[166] In an extensive open water body, **$L = \mathrm{sqrt}(\Delta x^2 + \Delta y^2)$**. In a narrow channel, **L** is the distance along the central axis of the channel. In estuaries and other constrained open water bodies, **L** is distance along the line of fastest propagation of tidal frequency waves, which includes some elements of both definitions, and which may vary between flood and ebb or between tidal waves of varying magnitude. Some hydrodynamicists (e.g. **Sobey 2000**) follow this convention of rotating the coordinate grid to follow the primary axis of flow, but typically use **x** for location and **Δx** for distance rather than **L**.

[167] Once water begins to move, it acquires inertia (momentum), which keeps it moving until retarded by friction, water surface slope in the other direction, or other force. Therefore, water surface slope indicates force and acceleration rather than velocity.

within an Estuary, Channel, or Marsh respectively) are useful for describing boundary conditions for studies of spatial-temporal patterns in adjacent study areas.

At the horizontal limits of any water body, there is a band of substrate that is alternately wetted (inundated) and dried (exposed, drained) as the water surface rises and falls (see Chapter Two). If the surface does not rise and fall simultaneously as a horizontal plain, then the temporal pattern of inundation and exposure (= "hydroperiod") will be spatially specific. In this band of alternately wet and dry conditions (the "coastal zone"), $\mathbf{Z(t)}$ is not rigorously defined for those times when the substrate surface is dry (i.e. for $\mathbf{t{:}Z_{XY}(t)}<\mathbf{z_{B,XY}}$). If calculations require a value of $\mathbf{Z(t)}$ for all t, then plausible options when there is no water depth include defining $\mathbf{Z_{XY}(t)}$ as 1) the substrate elevation $\mathbf{z_{B,XY}}$ or as 2) the "water table" ($\mathbf{Z_{WT}(t)}$) in the soil (see Chapter Eight).

I introduced geodetic datums in Chapter Two. Figure 55 combines the geodetic height relationships seen in Figures 22 and 23 with the tidal datums in Figure 11. In addition, I have included an additional set of tidal "means", equivalent to datums but calculated for the time period in question, and a set of "basic" heights relating the primary elevations that will be discussed throughout this thesis. The essential lessons of Figure 55 are that all elevations/heights are relative to datums, and that there are no completely stable datums that can be determined from surface features alone. Global mean sea level changes over time, crustal elevations (and hence benchmarks) move vertically over time, and understanding their relationship requires a datum independent of either. Unfortunately, while a purely mathematical ellipsoid surface can be defined, distances from it cannot be measured without tools dependent on the existing surface, as even GPS satellite orbits decay and must be recalibrated based on ground station geometry (NGS 2000).

B. Estuarine Stage & Tides

The most significant attributes of stage as a spatio-temporal variable in tidal estuaries[168] are 1) changes in stage are typically dominated by the tides, which propagate as long-period waves from the ocean up through the estuary; 2) up those branches of the estuary that become tidal and then non-tidal rivers, these long waves become dissipated by friction, turbulence, etc.; 3) in estuarine branches without tidal rivers, and in the lower reaches of tidal rivers, tidal waves may interact with the basin to show amplification of stage rather than attenuation; 4) tidal waves are essentially harmonic in form when they enter most estuaries, but their wave forms become increasingly distorted as they propagate; 5) stage at a location ($\mathbf{Z_{XY}(t)}$) varies over time in response both to tidal (harmonic) and non-tidal (non-harmonic) influences; 6) stage at a time $\mathbf{(Z_t(X,Y))}$ varies over space in response to distance-induced phase differences for tidal-frequency waves, in response to distance up-stream along gradients of mean fluvial water surface slope, and in response to other spatially-varying phenomena (wind fields, etc.); 7) $\mathbf{Z_{XY}(t)}$ can therefore differ significantly within (and between) estuaries; and 8) these differences between temporal patterns at different locations show significant, identifiable patterns. Notation to describe these attributes will be presented in this section.

Figure 56 summarizes a hierarchy of terms to describe the progressive changes that are seen in tidal waves as they propagate up-estuary. The simplest case is where tidal-frequency waves propagate upstream through an estuary from a single connection with the

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[168] See Chapter Two for discussion of estuaries. While estuaries are generally characterized as "tidal", some major rivers enter the sea in areas of little tidal action (e.g. Florida Bay), and the resulting estuaries are delineated more by their salinity gradients than their tidal amplitude gradients. The arguments presented here generally hold here, with the exception that temporal variation in these sites may be slight relative to that of other variables.

tidal "ocean" until they are completely attenuated by friction, turbulence, etc.[169]. This net attenuation with distance upstream is always accompanied by the passage of time (i.e. tidal phase is increasingly lagged behind the tidal source), by varying attenuation per unit distance (perhaps including local zones of range amplification), and by changes in wave form.

Because the tidal waves propagate unidirectionally upstream, and because this propagation is very slow relative to the passage of the moon overhead, increasing distance upstream ($\mathbf{L_{xy}}$) from the tidal source in this simple case is associated with an increasing time interval between lunar transit and the arrival of wave maxima ($\mathbf{HWI_{XY}}$ = High Water Interval) and minima ($\mathbf{LWI_{XY}}$ = Low Water Interval). This increasing time interval relative to HWI or LWI at the estuary mouth can be designated $\mathbf{\Delta HWI_{XY}}$ or $\mathbf{\Delta LWI_{XY}}$[170], and is often called the "tidal lag". Because wave propagation is slower in shallow water, as discussed in Chapter Two, $\mathbf{HWI_{XY}} < \mathbf{LWI_{XY}}$. Neither of these lags is consistent at any location for the entire tidal wave spectrum, especially where non-tidal variables also influence stage, but instead represent mean values for long data sets.

Although lag increases monotonically upstream from the tidal source, the diminution of tidal amplitude and influence with distance upstream is more complex, and can be characterized by three essentially independent changes. First, by definition, distance upstream is associated with a progressive increase in local MSL relative to any constant vertical datum ("mean water slope"), although this slope ($\mathbf{dMSL_{z0}/dL}$) is not constant. Second,

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[169] This case essentially describes the San Francisco Estuary. For sites where multiple connections exist to tidal sources (including some areas within the upper San Francisco Estuary, relative to down-estuary tides), the hydrodynamics are more complicated, but the mechanisms are consistent and the suggested notation can be easily adapted to local needs.

[170] $\mathbf{\Delta HWI_{O,XY}}$; $\mathbf{\Delta HWI_{PS,XY}}$; $\mathbf{\Delta HWI_{GG,XY}}$; and/or $\mathbf{\Delta HWI}$ may all be used where needed to avoid ambuiguity.

distance upstream is generally associated with decreasing tidal range ("attenuation" or "dampening"), although this is neither at a constant rate nor even necessarily monotonic (see Chapter Six for a comparison of distance upstream vs tidal range in the San Francisco Estuary). Where tidal range locally increases from the tidal source, it is termed "tidal amplification". Finally, the harmonic wave form is changed as it propagates ("distortion"), both because the faster motion of wave maxima than minima induces an increasing ebb-flood asymmetry in $\mathbf{Z(t)}$ vs $\mathbf{t}$ ("steepening"); and because bathymetry, wind, and other influences discussed in Chapter Two also alter the form of the $\mathbf{Z(t)}$ vs $\mathbf{t}$ curve ("deformation"). Together, changes in mean stage, tidal range, and tidal wave form can be summarized as up-estuary "tidal divergence"[171]. For a constant HW lag ($\mathbf{\Delta HWI_{XY}} = \mathbf{HWI_{XY}} - \mathbf{HWI_{PS}}$), divergence can be represented $\mathbf{\Delta Z'_{XY,O}(t_n)} = \mathbf{Z_{XY}(t_{n+\Delta HWI})} - \mathbf{Z_O(t_n)}$ for n = 0 (tidal maximum at the ocean) to n = 24.84 hrs (the next tidal maximum at the ocean). As with lag, divergence represents typical conditions at a location.

One consequence of the interaction of mean water slope and tidal attenuation/ amplification is the observation that, while local MLW or local MLLW generally rise monotonically with distance upstream, local MHW or MHHW can demonstrate a local maximum in mid-estuary and an upper-estuary minimum (see Chapter Six), before rising again as the river rises into its watershed.

1. Tidal and Non-tidal Influences on Stage

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[171] The word "divergence" is chosen explicitly to recognize the Newtonian convention that $\mathbf{Z(t)}$ in open oceanic water closely approximates the tides, and that tidal regimes in other locations represent alterations of this "essentially" tidal form. Equally logically, $\mathbf{Z(t)}$ in rivers above the zone of tidal influence could be treated as the "essential" pattern over time, with tides representing an alteration. Where no conceptual priority is intended, the word "difference" is used instead.

In estuaries and over marshes, $\mathbf{Z_{xy}(t)}$ is determined in part by tidal force fields (generated by the relative positions, motions, and interactions of the Sun, Earth, and the Earth's Moon[172]), and in part by terrestrial influences at many spatial scales. The contributions of Tidal forces and Non-Tidal influences on $\mathbf{Z_{xy}(t)}$ can be represented by $\mathbf{dZ(t)/dt = f(T, NT)}$. Tidal force fields can be accurately modeled and predicted as harmonic series using standard astronomical constants, and thus tides (defined at specific locations as the vertical movements over time caused by tidal forces) and the resulting time-dependent variable $\mathbf{T_{xy}(t)}$ or **T(t)** are routinely predicted using harmonic analysis based on astronomical time constants. The resulting Harmonic Predictions can be represented as $\mathbf{HP_{XY}(t)}$ or **HP(t)**[173]. Harmonic prediction residuals (**HR(t)**) are then defined by the difference between predictions and observations:

$$\mathbf{(10)\qquad HR(t) = Z(t) - HP(t)}$$

Non-tidal influences on **Z(t)** can include non-tidal force fields (e.g. barometric pressure, wind), and other variables including river flow, changes in local mean sea level due to ENSO or other changes in marine circulation, water density (temperature and salinity), etc. Non-tidal influences on **Z(t)** are too varied and too irregular[174] for comprehensive harmonic

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[172] Although other celestial bodies exert tidal influences on water at the Earth's surface, their influences are all insignificant due to their low masses and/or great distances from Earth.

[173] In this thesis I will not provide a rigorous distinction between "prediction" and "forecast", but I will follow convention that a prediction of a geophysical variable (e.g. daily tidal range) has a higher degree of certainty over a relatively long time horizon than a forecast (e.g. stage; see Chapters Three, Five, and Six).

As with **Z(t)** or **D(t)**, the **HP(t)** notations assume an unambiguous vertical datum.

[174] While some non-tidal influences on estuarine stage, such as mean monthly river flow, may demonstrate relatively predictable annual patterns, they are not harmonic (sinusoidal). Other influences, such as ENSO, storm surges, or floods, may recur at statistically-consistent intervals, but do not have consistent periodicities or sinusoidal forms.

analysis. Therefore, **Z(t)** cannot be accurately predicted for more than a few hours in the future[175].

Tidal and non-tidal influences on **Z(t)** in shallow waters and/or enclosed embayments can act independently, in which case their effects are additive. River deltas and other estuaries with little or no tidal action (e.g. the Neva at St. Petersburg; the Indian River in Florida) demonstrate substantial variation in **Z(t)** in response to weather-induced changes in runoff as well as to more local weather conditions (wind, barometer). By definition, there are no estuaries in which the only influences on **Z(t)** are tidal. However, **Z(t)** in the non-estuarine coastal zone is highly dominated by tides, and the non-tidal influences that occur (ENSO-induced changes in regional sea level; oceanic storm surges; etc.) generally shift the entire tidal stage profile up or down, while preserving in large part its daily range, timing, and form. Consistent interactions (energy dispersion, resonance, etc.) between tidal force fields, tides in the adjoining ocean (= boundary conditions), and embayment form and bathymetry can generate consistent site-specific "shallow-water tides" (patterns in **Z(t)** that are in phase with tidal frequencies or small integer multiples of them). These patterns are generally amenable to harmonic analysis and prediction.

In contrast, significant non-additive tidal/non-tidal interactions in shallow water, such as changes in the propagation of tidally-induced low-frequency waves through a water body as its mean daily stage varies with runoff or ENSO, preclude rigorous disaggregation of "tidal" and "non-tidal" changes in water level (for "tides" and "tidal" as defined above). Therefore, while it may be possible to measure and predict oceanic tides ($\mathbf{T_O(t)}$), it is

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[175] A corollary which will be elaborated later is that tidal harmonic prediction residuals (**HR(t)**) cannot be completely resolved by harmonic analysis either.

impossible to rigorously measure or predict “estuarine tides” ($\mathbf{Z_{ET}(t)}$), or their corollaries “marsh tides” or “over-marsh tides” ($\mathbf{Z_{MT}(t)}$). However, the use of “tide”, “tides”, and “tidal” to describe $\mathbf{Z(t)}$ in estuarine settings has too widely used for too long to be entirely rejected. Instead, rigorous definitions of and distinctions between “estuarine tide” and “estuarine stage” are mandatory for rigorous discussion and analysis. Therefore, for the remainder of this thesis, “estuarine tide” and “marsh tide” are generally used to represent those variations in stage ($\mathbf{dZ(t)/dt}$) that correspond to tidal frequencies ≤ 25 hours, but explicit qualifications and definitions are added when needed for clarity or precision.

Despite site specificity and variations over time in the forcing functions, $\mathbf{Z(t)}$, $\mathbf{HR(t)}$, and $\mathbf{\Delta Z(t)}$ in estuaries all demonstrate a number of repetitive patterns, and some of these patterns were noted before. For example, $\mathbf{Z(t)}$ is dominated in most estuaries by the tides, and the most obvious repetitive patterns are those associated with tides in the nearby coastal ocean. Because tides propagate as progressive waves, $\mathbf{Z(t)}$ at any point in an estuary lags behind $\mathbf{Z_O(t)}$; the lag increases with distance up-estuary; and the lag is greater at any location for stage minima than for maxima. In general, the range of tidal-frequency variations in $\mathbf{Z(t)}$ (tidal waves)decreases with distance up-estuary (muting). With some combinations of estuarine basin size, shape, and bathymetry, tidal waves can resonate in the estuary (seiche) and increase the range of tidal-frequency variation in $\mathbf{Z(t)}$ at some locations.

Other patterns are site specific or simply not widely recognized. For example, the western portion of the San Francisco Estuary (South Bay, Central Bay, San Pablo Bay) is characterized by a standing wave, while tidal range in the northern and eastern portion (including San Pablo Bay) is dominated by a progressive dampening up-river. Thus, San Pablo Bay is both amplified and attenuated, resulting in near-oceanic range tides. Also, all of the minor rivers entering the San Francisco Estuary display their greatest tidal ranges at some distance up-stream of their mouths, even though the ranges at their mouths diminish consistently with distance up-estuary (once north and east of the South Bay - San Pablo Bay standing wave). These patterns will be demonstrated and discussed in Chapter Six.

2. Tidal Datums and Tidal Means

Tidal datums were discussed in Chapter Two as a legally-recognized sub-set of calculated mean tidal heights and, as such, they are functions of location and time period. In this subsection, I will present a symbolism to clearly and efficiently indicate both official tidal datums and tidal means more generally.

Evaluation of any tidal mean values involves establishment of conventional definitions and practices, some of which were discussed in Chapter Two (see also Schureman 1929 and Marmer 1951). In summary, mean sea level (MSL) is a long-term average of regularly-spaced intervals (typically hourly; = $\mathbf{Z(t_m)}$, for $t_{m+1} = t_m + 1$ hr), while all other tidal datums are based on individuals tides (LW and HW). Mean tide level (MTL), which approximates MSL but which typically differs up-estuary, is the numeric mean of the average stage at high water ("Mean High Water" = "MHW") and the average stage at low water ("Mean Low Water" = "MLW"). In mixed semidiurnal tidal regimes, HW's generally alternate between Higher High Water (HHW) and Lower High Water (LHW) and LW's generally alternate between Lower Low Water (LLW) and Higher Low Water (HLW)[176], which allow four new tidal datums (MHHW, MLHW, MHLW, and MLLW), although only

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[176] Exceptions to the strict alternation between HHW and LHW, or between LLW and HLW, and implications for tidal means and datums, will be discussed in detail in Chapter Five.

MHHW and MLLW are used routinely out of this list. It should be clear from this discussion that all tidal datums are defined only for the site where the data were collected, although as a practical matter they are assumed to project horizontally (Marmer's "Tidal Datum Planes") for at least some distance. Wherever there is potential confusion, therefore, it is appropriate to use these notations with geographic subscripts (e.g. $\mathbf{MSL_{XY}}$, $\mathbf{MHHW_{XY}}$).

Because of their legal significance (delineating public vs. private property, navigation safety and liability, etc.) NOS and other governmental entities calculate and publish official tidal datums at individual locations on the basis of specified tidal epochs and specified absolute vertical datums (NGVD, NAVD88, "local", etc.). Where NOS references datums for a site as "accepted" but does not indicate the datum used in their collection, a good notation indicates the location, the authority, and the reference datum: $\mathbf{MLLW_{GG,\,NOS,\,NGVD}}$. Where the epoch is indicated, it is more informative to indicate the beginning and ending years of the epoch: $\mathbf{MSL_{GG,\,1960\text{-}78,\,Local}}$. If longer time periods have been collected, it is sometimes also useful to indicate a datum for a period longer than a 19 year epoch: $\mathbf{MHHW_{GG,\,1897\text{-}1999,\,NGVD}}$. Finally, where evaluations of long-term change are made, it is sometimes necessary to describe a datum calculated over one time period in terms of one from another (usually "accepted") period: $\mathbf{MSL_{GG,\,1854\text{-}1999,\,MLLW\ NOS}}$. In these cases, the accepted or reference datum is treated as an absolute vertical plane. In addition to long-term means, monthly tidal means can be useful for evaluating and comparing seasonal patterns. In this case, the month or months that are evaluated are specified in addition to the period, location, and reference datum (e.g. $\mathbf{MLW_{GG,\,Jan\ 1980\text{-}99,\,MLLW\ 1980\text{-}99}}$).

3. Tide Prediction and Stage Forecast

Because stage reflects both tidal and non-tidal influences, harmonic prediction residuals[177] also reflect both tidal and non-tidal influences, and therefore do not represent the theoretical "non-tidal stage" $\mathbf{Z_{NT}(t)}$ that would have occurred in the absence of tidal force fields. While up-estuary Stage Divergences

(11) $$\mathbf{\Delta Z_{XY,O}(t) = Z_{XY}(t) - Z_{O}(t)}$$

differ from $\mathbf{HR_{XY}(t)}$ in that their calculations include the residuals associated with imperfect harmonic prediction at the oceanic site, they similarly include both tidal and non-tidal influences, and are thus not completely resolvable harmonically.

I noted in Chapter Two that harmonic predictions in estuaries are typically based on a single "reference station" or "primary station", which is usually near the estuarine mouth and which usually has a long series of stage measurements. Stage/tide predictions for other ("secondary") stations can be generated either by modifying $\mathbf{HP_{PS}(t)}$ with tidal correctors (these are then denoted $\mathbf{HPC_{SS}(t)}$) or by derivation of tidal harmonic constants at the secondary stations ($\mathbf{HP_{SS}(t)}$). In the first case, the residual is

(12) $$\mathbf{HR_{SS,C}(t) = Z_{SS}(t) - HPC_{SS}(t)}.$$

In the second, the residual is

(13) $$\mathbf{HR_{SS,H}(t) = Z_{SS}(t) - HP_{SS}(t)}.$$

In the great majority of cases, $\mathbf{HP_{XY}(t)}$ in an estuary for any location other than the Primary Station(s) is really $\mathbf{HPC_{SS}(t)}$, which is generally based on the site(s) in the estuary

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[177] Although the convention has been to define $\mathbf{HR(t)}$ as the difference between observed and predicted (Equation 10), residuals derived from quotients ($\mathbf{Z(t)/HP(t)}$), or exponential or functions could be defined. None of the discussion in this section would be significantly changed.

with the longest data set on **Z(t)** and adapted to other xy locations through the use of additive or multiplicative “Tidal Reducers”

(14) $$\mathbf{HP_{XY}(t) = HP_{PS}(t) + TRA_{XY}}\text{; or}$$

(15) $$\mathbf{HP_{XY}(t) = HP_{PS}(t) \times TRM_{XY}},$$

which serve as a simple approximation to divergence. NOS publishes both high water and low water lags and reducers for the San Francisco Estuary. In addition, it was noted before that NOAA has recently issued tidal harmonic constants for secondary stations throughout the San Francisco and other estuaries, wherever long enough time series of **Z(t)** data have been collected. Thus, direct calculation of $\mathbf{HP_{XY}(t)}$ is now possible at multiple locations throughout the Estuary. However, this approach does not incorporate non-harmonic influences. Also, because the time series used are much shorter than those at established primary stations, and because the new stations are almost always further up-estuary than the established primary stations (implying greater non-tidal and therefore non-harmonic influences), the new harmonic predictions may actually be less accurate than using the standard method described above.

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